Exercise 304

Iodine-131 is a radioactive substance that decays according to the function $Q(t) = Q_0 \cdot e^{-0.08664t}$, where Q_0 is the initial quantity of a sample of the substance and t is in days. Determine how long it takes (to the nearest day) for 95% of a quantity to decay.

Solution

If 95% of a quantity decays, then only 5% of it is left.

$$Q(t) = Q_0 \cdot e^{-0.08664t}$$

$$0.05 Q_0 = Q_0 \cdot e^{-0.08664t}$$

Divide both sides by Q_0.

$$0.05 = e^{-0.08664t}$$

Take the natural logarithm of both sides.

$$\ln 0.05 = \ln e^{-0.08664t}$$

Use the property of logarithms that allows the exponent of the argument to be brought down in front.

$$\ln 0.05 = (-0.08664t) \ln e$$

Use the fact that $\ln e = 1$.

$$\ln 0.05 = -0.08664t$$

Solve for t by dividing both sides by -0.08664.

$$t = -\frac{\ln 0.05}{0.08664} \approx 34.6$$

Therefore, it takes about 35 days for 95% of a sample of Iodine-131 to decay.